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ABSTRACT

The purpose of this study was to characterize and compare novice and experienced elementary teachers' pedagogical knowledge and pedagogical content knowledge of three major topics in mathematics: whole number operation, fractions, and geometry. The study participants were 26 preservice elementary teachers and 28 experienced kindergarten through sixth grade teachers. Data were collected via the Survey on Teaching Mathematics, a research-designed instrument that assists in describing pedagogical content knowledge. The results seemed to indicate that experienced teachers possess a greater conceptual understanding of whole number operations than do novice teachers, but that both novice and experienced teachers possess primarily a procedural knowledge of fractions. In addition, the results indicated that both novice and experienced teachers think that a good teacher is one who shows and tells students how to do the work. The findings imply that both novice and experienced teachers need to revisit and extend their own mathematical understandings. They need to explore, identify, and challenge their assumptions about the teacher's role and to develop pedagogical content knowledge. (Contains 11 references.) (Author/ND)

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Elementary Teachers' Pedagogical Content Knowledge of Mathematics

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Abstract

The purpose of this study was to characterize and compare novice and experienced elementary teachers' pedagogical knowledge and pedagogical content knowledge of three major topics in mathematics: whole number operations, fractions, and geometry. Twenty-six preservice elementary teachers and twenty-eight experienced kindergarten through sixth grade teachers participated in this study. Data were collected via the Survey on Teaching Mathematics (Rich, Lubinski & Otto, 1994), which is a researcher-designed instrument that assists in describing pedagogical content knowledge. The results seem to indicate that experienced teachers possess a greater conceptual understanding of whole number operations than do novice teachers, but that both novice and experienced teachers possess primarily a procedural knowledge of fractions. In addition, the results indicate that both novice and experienced teachers think that a good teacher is one who shows and tells students how to do the work.

Elementary Teachers' Pedagogical Content Knowledge of Mathematics

When it was in its infancy, teacher education primarily focused on a teacher's knowledge of subject matter content (Shulman, 1986). However, for the past decade or more, teacher education research has emphasized the effectiveness of general pedagogical methods independent of subject matter content, such as how teachers manage their classrooms, organize activities, allocate time and turns, structure assignments, ascribe praise and blame, formulate the levels of their questions, plan lessons, and assess student understanding (Ball & McDiarmid, 1990; Shulman, 1986, 1987, 1988; Onslow, Beynon, & Geddis, 1992). In addition to teachers' subject matter (content) knowledge and their knowledge of general instructional methods (pedagogical knowledge), Shulman (1986, 1987, 1988) has suggested that teaching expertise should be described and evaluated in terms of pedagogical content knowledge. According to Shulman (1986), pedagogical content knowledge

> include[s] . . . the most useful forms of representation of . . . ideas, the most powerful

> analogies, illustrations, examples, explanations, and demonstrations - in a word, the ways of representing and formulating the subject that make it comprehensible to others . . . [It] also includes an understanding of what makes the learning of specific concepts easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning (p. 9)

Pedagogical content knowledge is the synthesis or integration of teachers' subject matter knowledge and their pedagogical knowledge into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction (Gudmundsdottir, 1987; Shulman, 1986, 1987, 1988). It is that form of knowledge that makes teachers teachers rather than subject area experts, for teachers differ from biologists, historians, writers, or mathematicians, not necessarily in the quality or quantity of their subject matter knowledge, but in how that knowledge is organized and used. The teaching process requires teachers to "transform" their subject matter knowledge for the purpose of teaching

(Gudmundsdottir, 1987; Shulman, 1986, 1987). This transformation occurs as the teacher engages in the act of "pedagogical reasoning", i.e. examines and critically interprets instructional materials in terms of the teacher's own understanding of the subject matter; thinks through the key ideas and identifies alternative ways of representing them to students as analogies, metaphors, examples, demonstrations, simulations, etc.; adapts the material to students' characteristics such as ability, gender, language, culture, prior knowledge, conceptions, misconceptions, expectations, difficulties, strategies, etc.; and finally tailors the material to the specific students in a classroom (Shulman, 1987).

Studies indicate that novice teachers have major concerns about pedagogical content knowledge and that they struggle with how to transform and represent concepts and ideas in ways that make sense to the specific students they are teaching (Ball & Wilson, 1990; Borko et al., 1992; Onslow, Beynon, & Geddis, 1992). Onslow, Beynon, and Geddis (1992) described the developing pedagogical content knowledge of two student teachers enrolled in a one-year elementary teacher education program, by focusing on a dilemma faced by the student teachers as they attempted to transform their

understanding of a topic in mathematics into a form that could be understood by their students.

At this point in their career, student teachers' understanding of pedagogy is gradually being molded to fit a style of teaching with which they feel comfortable. Often the style of teaching advocated by university faculty is in conflict with the style of teaching remembered by student teachers during their own schooling (Onslow, Beynon, & Geddis, 1992). Many student teachers remember mathematics classrooms in which the teacher tells and the students remember. Taking on the role of facilitator, helping learners to construct conceptual knowledge, emerges as an exciting and rewarding alternative. This style of teaching, however, involves the transformation of subject matter knowledge, and so requires a firm grasp of various components of pedagogical content knowledge.

Herein lies the student teachers' dilemma. When novice teachers become frustrated with the difficulties inherent in teaching mathematics meaningfully, the time constraints involved in covering the content of the curriculum, and the need to cope with individual differences, they often resort to teaching the way they

were taught despite their desire to do otherwise. One student teacher wrote:

> I found that it was difficult to compromise my new views about presenting mathematical concepts with my old memories about how I learned the same material. Perhaps I was surprised and even a little annoyed at how much my own education may influence how I now educate (or attempt to educate!) others. I intended to let or allow students to discover or develop the concept . . . on their own as much as possible. What interfered with this intent were time constraints and the relative difficulty of this approach as opposed to simply presenting the "rules" for students to follow and apply. (Onslow, Beynon, & Geddis, 1992, p. 308)

Similar results were found by Borko et al. (1992), who focused on a single episode in one student teacher's experiences. When confronted with a student's question that required a conceptual explanation, the student teacher attempted to provide a concrete example. However, she made an error and ultimately decided to abandon the attempt, though she believed that good mathematics teaching primarily involved making

mathematics relevant and meaningful for students. She then focused on computational procedures by demonstrating the use of the algorithm and providing guided and independent practice.

This concern about pedagogical content knowledge is present even in new teachers who possess substantial subject matter knowledge. Ball and Wilson (1990) focused on the underlying assumptions of "alternate route" teacher certification programs. First, it is assumed that college graduates who majored in liberal arts have, in general, more subject matter knowledge than do their teacher education counterparts, since they have received an education uncluttered with professional course work. Second, it is assumed that subject matter knowledge is the only professional knowledge one needs to acquire in formal university or college settings. Other types of knowledge necessary to teaching, e.g., pedagogical knowledge or pedagogical content knowledge, can and should be acquired through practical experience as a full-fledged teacher.

In order to examine these assumptions, Ball and Wilson (1990) collected data on two groups of novice secondary mathematics teachers. The first group consisted of 22 undergraduate students preparing to

teach and majoring in mathematics at Dartmouth College, Illinois State University, and Michigan State University; the second group, 21 postbaccalaureate mathematics majors entering teaching through the Los Angeles Unified School District alternate route program. Little difference was found in the mathematical understandings between novices in the alternate route program and teacher education candidates, either when they entered their program or when they finished. These results not only revealed that both groups possessed minimal knowledge of elementary topics, but also that the novice teachers were often aware that all they had learned were rules and procedures, which they had memorized and learned to use in algorithmic ways. No one had helped them develop meaningful understandings of the rules and procedures. One of the teacher education mathematics majors remarked:

> . . . here I am supposedly this math wizard and I've got a lot of knowledge and I've probably made more connections than a lot of people, but there are a lot of connections that I haven't made. I haven't seen these things before and I don't know where I was supposed to learn them -- in high school? Or middle school? (p. 11)

To analyze the validity of the second assumption two aspects of the novice teachers' developing ideas about teaching were examined: their notions about the teacher's role in helping students learn about mathematics, and their pedagogical perspective on the content. As for role, Ball and Wilson (1990) saw little difference in the teachers' views as a group. When the teachers entered their programs, both groups thought that a teacher who shows and tells students exactly how to do the work is most likely to help students learn mathematics. By the end of the program, more novices in both groups had shifted to describing good teaching in terms of "leading" and "guiding" students rather than telling. But in neither group did more than one or two people favor a more facilitative, constructivist-oriented style.

Ball and Wilson (1990) saw similar trends in responses to interview questions which posed pedagogical problems (e.g., a student suggests a nonstandard algorithm, asks a question, presents an error on a paper) and asked respondents how they would deal with the situation. In every case, teacher candidates and teacher trainees alike said they would respond directly to the student, telling the student if the idea was

correct, showing him or her what to do, answering questions directly. And there was virtually no change in this over the course of their programs. Ball and Wilson (1990) claim that teachers who themselves are tied to a procedural knowledge of mathematics are not equipped to represent mathematical ideas to students in ways that will connect their prior and current knowledge and the mathematics they are to learn, a critical dimension of pedagogical content knowledge.

While this literature indicates that novice teachers have major concerns about pedagogical content knowledge and that they struggle with how to transform and represent concepts and ideas in ways that make sense to the specific students they are teaching (Ball & Wilson, 1990; Borko et al., 1992; Onslow, Beynon, & Geddis, 1992), the assumption that pedagogical knowledge or pedagogical content knowledge can be acquired through practical experience warrants further investigation.

The purpose of this study was to characterize and compare novice and experienced elementary teachers' pedagogical knowledge and pedagogical content knowledge of three major topics in mathematics: whole number operations, fractions, and geometry.

Method

Participants

Twenty-six preservice elementary teachers and twenty-eight experienced kindergarten through sixth grade teachers participated in this study. The preservice teachers were elementary education majors at Illinois State University. The experienced teachers are employed in eight schools located in or near Bloomington, Illinois. The schools include one Catholic school and seven public schools.

All teachers in the sample self-selected to participate in a five-year research project that provided information about mathematics, mathematics learning, and mathematics teaching (C. Lubinski, A. Otto, & B. Rich, Influences on Preservice Teachers' Instructional Decision Making, 1992-1997). Extending pedagogical content knowledge was a major focus of the project.

Data Collection Procedures and Instrument

Baseline data regarding pedagogical content knowledge were collected by the staff of the research project in which the teachers participated. This data constituted the data set for the study. Data for each teacher were collected using the Survey on Teaching

Mathematics (Rich, Lubinski, & Otto, 1994), a researcher-designed instrument that assists in describing pedagogical content knowledge. The survey consists of 12 questions primarily involving whole number operations, fractions, geometry, number sense, and mathematical reasoning. The questions, which focus on the instructional decisions a teacher would make in regard to specific classroom situations involving mathematics, are intended to extract information regarding the participants' own knowledge and beliefs about mathematics.

Analysis of the Data

The data were analyzed using qualitative research methods. The teachers' pedagogical content knowledge was analyzed using Shulman's (1987) description of "pedagogical reasoning." Recall that the process of pedagogical reasoning involves comprehension and transformation. According to Shulman (1987), teachers much first comprehend the content of instruction and then transform the comprehended ideas in some way for the purpose of teaching. As a result, the teachers' interpretations, representations, and adaptations of the content were examined. It appears that teachers who possess only a procedural knowledge of mathematics are

unable to transform and represent mathematical concepts and ideas in ways that make sense to their students (Ball & Wilson, 1990; Borko et al., 1992; Onslow, Beynon, & Geddis, 1992). Therefore, the participants' interpretations, representations, and adaptations of the content were analyzed using qualitative research methods and the responses coded in order to determine the degree to which their knowledge was procedural.

The teachers' pedagogical knowledge was analyzed in terms of their approach to teaching mathematics as identified by Kuhs and Ball (1986). In the learner-focused approach to teaching mathematics, the teacher's role is to stimulate student learning by posing problems, designing experiences, and asking questions and to facilitate student learning by listening, probing, accepting, restating, and encouraging. The learner actively participates in the exploration of ideas, i.e. the learner is a creator of mathematics. In the content-focused with emphasis on understanding approach to teaching mathematics, the teacher's role is both to organize the content and to guide student learning. The learner is considered to be a discoverer of the mathematics presented by the teacher via the problems posed for investigation. In the content-

focused with emphasis on performance approach to teaching mathematics, the teacher's role is to present material in an expository style, explaining concepts and demonstrating skills. The learner listens, responds to teacher questions, and does exercises using procedures that have been modeled by the teacher, i.e. the learner's role is to imitate the teacher.

Results

Given below are the results of one question from each of the following topics: whole number operations, fractions, and geometry.

Whole Number Operations

#1. (From NCTM, 1991) One day your students finish working on addition and subtraction with regrouping. On a written test, many of them "forget" to regroup when they need to in subtraction. Instead they do this:

$$\begin{array}{r} 60 \\ -28 \\ \hline 48 \end{array}$$

a) What would you do and why?
b) Why is this an appropriate thing to do?

Fifteen of the preservice teachers and four of the experienced teachers focused exclusively on the procedure. A common response for this group of teachers was to "review how to subtract from back to front with a 0. Tell them to make the zero into a ten, turn the 6 into a 5, and then subtract." These

teachers' approaches to teaching mathematics seem to fit with the content-focused with emphasis on performance approach described by Kuhs and Ball (1986), since most responded that they would tell or show the students what to do. Nearly all of the teachers in the study interpreted this as a "straightforward" regrouping problem. Only two of them, both preservice teachers in this group, responded directly regarding their interpretation of the content. One teacher claimed that "we . . . show that the zero in 60 is a 10 and 8 have to be taken away from it." The other responded that "I'll illustrate how to make the zero a 10 and the 6 a 5. I'll explain to them how the 10 is borrowed from the 6." The teachers in this group tended to rely primarily on the symbolic or algorithmic representation of the content. However, one experienced teacher did indicate that he or she would also "have them tell me if the answer is reasonable."

Five of the preservice teachers and 13 of the experienced teachers provided both procedural and conceptual responses to this question. It is interesting to note that all of the experienced teachers in this group except two said that they would "go back to manipulatives" first and "then bring [the students]

back to pencil and paper," whereas only one of the preservice teachers expressed this idea and in fact, three of them said that they would "review the process" and then "try hands-on methods" if necessary. One preservice teacher claimed that using base ten blocks "gives the child a physical explanation of why you cancel the six and make the zero a ten." While all of the teachers in this group mentioned the use of manipulatives as a way to represent the content in addition to the algorithm, four of the preservice teachers and ten of the experienced teachers responded that they "would [use the manipulatives to] model the procedure." One experienced teachers also mentioned the use of a problem solving context. Nevertheless, this approach to teaching mathematics also seems to fit with the content-focused with emphasis on performance approach (Kuhs & Ball, 1986), since the teacher is the one who is actually doing the manipulating. Just one preservice teacher and three experienced teachers in this group mentioned having the students use the manipulatives, either to do more problems involving regrouping or to check the problem above, "because it allows the student to discover his/her own error." Though only the preservice teacher mentioned the use of

cooperative learning groups, these teachers' approaches to teaching mathematics therefore seem to fit with the content-focused with emphasis on understanding approach described by Kuhs and Ball (1986).

Six of the preservice teachers and eight of the experienced teachers gave only conceptual responses to this question. Like the previous group, almost all of the teachers in this group mentioned the use of manipulatives as a way to represent the content. Three of the preservice teachers and two of the experienced teachers said something like, "I would get out the [manipulatives] and demonstrate over how to do this problem and more like it. Evidently my first demonstration was too abstract. It needs to be more visual." Again, this approach to teaching mathematics seems to fit with the content-focused with emphasis on performance approach (Kuhs & Ball, 1986), since the teacher is the one using the manipulatives and the students are only observers. Two preservice teachers and five experienced teachers in this group mentioned having the students use the manipulatives instead, because "they would understand why they need to regroup." Of these, one preservice teacher and one experienced teacher mentioned the use of cooperative

learning groups and one experienced teacher mentioned the use of a problem solving context. Rather than using manipulatives, one experienced teacher's conceptual explanation involved the use of estimation in a problem solving context. These teachers' approaches to teaching mathematics seem to fit with the content-focused with emphasis on understanding approach described by Kuhs and Ball (1986). Just one participant in the entire study, a preservice teacher in this group, mentioned having the "students . . . explain their thinking to better understand where and why they were making this mistake." Thus this teacher's approach to teaching mathematics seems to be the only one to fit with the learner-focused approach described by Kuhs and Ball (1986).

Finally, three of the experienced teachers' responses could not be categorized as either procedural or conceptual. One teacher simply stated that he or she would "go back and reteach with different strategies." Another teacher in this group said that, "I may go on to another unit and come back to it (subtraction) later as the students are obviously not ready for subtraction with regrouping." The third teacher discussed how he or she would assign grades for this test.

Fractions

#4. Suppose you are working with a group of students on addition and subtraction of fractions. Several of them are solving problems as shown below:

$$1/2 + 1/3 = 2/5$$

When asked about their solution they respond, "Jane made one out of two free throws in the first half and one out of three in the second. So, she made two out of five in the game."

a) What do you respond?

b) Why do you respond this way?

Thirteen of the preservice teachers and 11 of the experienced teachers focused solely on the procedure by relying primarily on the algorithmic representation of the problem, again taking an approach to teaching mathematics that seems to fit with the content-focused with emphasis on performance approach described by Kuhs and Ball (1986). This group provided four different interpretations of this problem. Ten preservice teachers and six experienced teachers gave responses involving or related to the idea that "when adding and subtracting fractions, you must find a common denominator." Two preservice teachers equated this with "shooting" the same "number of shots in each half." One preservice teacher stated that "the denominator must be the same because you are adding two parts of a whole and not two separate parts of two separate wholes." Similarly, one experienced teacher wrote "we need to

look at the total number of shots as our denominator so that one out of five and another one out of five gives us a total of two out of five." One preservice teacher and two experienced teachers commented that they "would explain the difference between ratios and fractions." One preservice teacher responded that "1/2 could also be used to represent three out of six or five out of ten [which] would not [give] the same answer." And finally, one experienced teacher gave the following explanation: "Jane made 1/2 of her free throws in the first half and 1/3 of her free throws in the second half. What fraction of her free throws did she make during the game? Did she make more than 1/2 of them or less than 1/2 of them." One preservice teacher and two experienced teachers in this group avoided interpreting the problem by offering comments like "I would have to agree that their logic makes sense, but I would suggest we take another look at the process of adding fractions and the steps we needed to take."

Four of the preservice teachers and four of the experienced teachers provided both procedural and conceptual responses to this question. In addition to using the algorithm, three of the preservice teachers and two of the experienced teachers in this group said

that they would draw pictures or use manipulatives to represent the problem, taking an approach to teaching mathematics that also seems to fit with the content-focused with emphasis on performance approach (Kuhs & Ball, 1986). The other three teachers in this group stated that they would have the students draw a picture or try using manipulatives in order to "lead them to discover you have to have a common whole," thereby taking an approach to teaching mathematics that seems to fit more with the content-focused with emphasis on understanding approach (Kuhs & Ball, 1986). This group also provided four different interpretations of the problem. Three preservice teachers and two experienced teachers claimed "that in order to add fractions, the denominators must be the same." One preservice teacher mentioned that "[fractions are] parts of a whole . . . they need to learn the difference between ratio and . . . fraction." One experienced teacher claimed "that this answer would show percentage rather than the solution to the fraction problem." Another experienced teacher said that he or she "would . . . show the student that . . . 1/2 is not necessarily one out of two but may be two out of four."

Four of the preservice teachers and eleven of the

experienced teachers gave only conceptual responses to this question. This time three preservice teachers and six experienced teachers in this group mentioned drawing pictures or showing with manipulatives, an approach to teaching mathematics that seems to fit with the content-focused with emphasis on performance approach (Kuhs & Ball, 1986). The remaining six teachers in this group suggested having the students draw pictures or use manipulatives in order to "find their mistakes," an approach to teaching mathematics that, as previously stated, seems to fit with the content-focused with emphasis on understanding approach (Kuhs & Ball, 1986). This group provided three different interpretations of this problem. One preservice teacher and four experienced teachers gave responses like "ratios work differently than fractions [which are] parts of a whole." One preservice teacher and one experienced teacher claimed that "we must combine like things." Two preservice teachers and five experienced teachers offered responses related to the idea of using the same whole. One experienced teacher in this group avoided interpreting the problem.

Five of the preservice and two of the experienced teachers' responses could not be categorized as either

conceptual or procedural. Four preservice teachers and one experienced teacher believed that the student's solution was correct. In fact, one preservice teacher commented, "If this is an easy way for students to remember how to add fractions, then I would use "fun" examples like this when I explain the addition of fractions." One preservice teacher admitted that he or she did not know how to respond, since "the logic is rational" but the answer is wrong. One experienced teacher responded mysteriously, "I wonder what Jane would say about you saying that she made only two out of five shots? I think she might be a bit upset. See if you can figure out why I am saying this."

Geometry

#5. (From NCTM, 1991) The following problem is posed to a group of students. "Suppose you had 64 meters of fence with which you were going to build a pen for your large dog, Bones. What are some different pens you could make if you use all the fencing? What is the pen with the lease play space? What is the biggest pen you can make - the one that allows Bones the most play space? Which would be the best for running?" After considering the problems, the students explain that it doesn't matter since all the pens will have the same perimeter - 64 meters.
a) Explain why they gave this response.
b) How would you respond to their solution?
c) Explain.

Just one experienced teacher provided a response to this question that was both procedural and conceptual,

since he or she was the only one that specifically mentioned an area formula. "I would first of all draw a picture showing the different areas that have a perimeter of 64 meters. I would point out how to find area. (length x width)." This teacher's approach to teaching mathematics seems to fit with the content-focused with emphasis on performance approach (Kuhs & Ball, 1986). Though all of the other teachers attempted only conceptual responses, 18 of the preservice teachers and seven of the experienced teachers mentioned that they themselves would draw pictures or use manipulatives to demonstrate, an approach to teaching mathematics which also seems to fit with the content-focused with emphasis on performance approach (Kuhs & Ball, 1986). One preservice teacher claimed that "an example from the teacher . . . lights up the bulb! Then the students are off and running." Eight of the preservice teachers and 18 of the experienced teachers instead suggested having the students engage in these sorts of activities, thereby taking an approach to teaching mathematics that seems to fit with the content-focused with emphasis on understanding approach (Kuhs & Ball, 1986). One experienced teacher did not respond to this question at all and one experienced teacher's response could not be

categorized. He or she stated that "the students didn't want to go to the work of figuring this problem."

There were three different interpretations given. Though 19 preservice teachers and 12 experienced teachers specifically mentioned something to the effect that "[the students] were confusing perimeter with area," 13 preservice teachers' and 15 experienced teachers' responses were limited to the consideration of rectangular pens only. One preservice teacher claimed that of these, the "square produces [the] most area." Five preservice teachers and one experienced teacher at least included the possibility of a circular pen and three preservice teachers and one experienced teacher also mentioned the use of other polygonal figures. One experienced teacher focused in part on definitions of perimeter and area, by "explain[ing] to them that area is the amount of space an object covers and the perimeter is the length and width of an object." Finally, two experienced teachers claimed that the solution was correct. For example, "While they are correct I would encourage them to look at . . . what would be an interesting and creative way to use an area." Only four teachers mentioned using cooperative learning groups.

Discussion

Slightly more than one-half of the preservice teachers' responses to both question one and four were procedural only, somewhat supporting the findings which seem to indicate that novice teachers possess primarily a procedural knowledge of mathematics (Ball & Wilson, 1990; Borko et al., 1992). The research results suggesting that novice teachers think that a good teacher is one who shows and tells students how to do the work (Ball & Wilson, 1990; Borko et al., 1992; Onslow, Beynon, & Geddis, 1992), were also supported, since more than two-thirds of the preservice teachers' responses to all of the questions fit with the approach to teaching mathematics that emphasizes performance (Kuhs & Ball, 1986).

Unlike the preservice teachers, very few of the experienced teachers gave strictly procedural responses to question one. The fact that three-fourths of them as opposed to only one-third of the preservice teachers suggested the use of manipulatives seems to indicate that experienced teachers possess a greater conceptual understanding of whole number operations than do novice teachers. However, many more experienced teachers (about the same fraction as preservice teachers - just

less than one-half) gave strictly procedural responses to question four. This seems to suggest that experienced teachers have a greater conceptual understanding of whole number operations than they do of fractions. In addition, about two-thirds of the experienced teachers' responses for questions one and four fit with the approach to teaching mathematics that emphasizes performance (Kuhs & Ball, 1986). Together these results seem to indicate that neither pedagogical knowledge nor pedagogical content knowledge is necessarily acquired through practical experience.

Though nearly all of the teachers provided conceptual responses to question five, the fact that a great many of their interpretations were limited to the consideration of rectangular pens only seems to suggest that their understanding of this topic is severely restricted. One wonders whether more procedural responses would have been given if the teachers had thought of other possibilities. One also wonders what effect this limited interpretation has on the teacher's approach to teaching mathematics, since more experienced teachers gave responses to this question that fit with the approach which emphasizes understanding than with the approach which emphasizes performance (Kuhs & Ball, 1986).

Implications

To become a good mathematics teacher requires thoughtful reflection. Both novice and experienced teachers, themselves the products of traditional mathematics classrooms, need to revisit and extend their own mathematical understandings. They need opportunities to explore, identify, and challenge their assumptions about the teacher's role, as well as to develop pedagogical content knowledge.

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